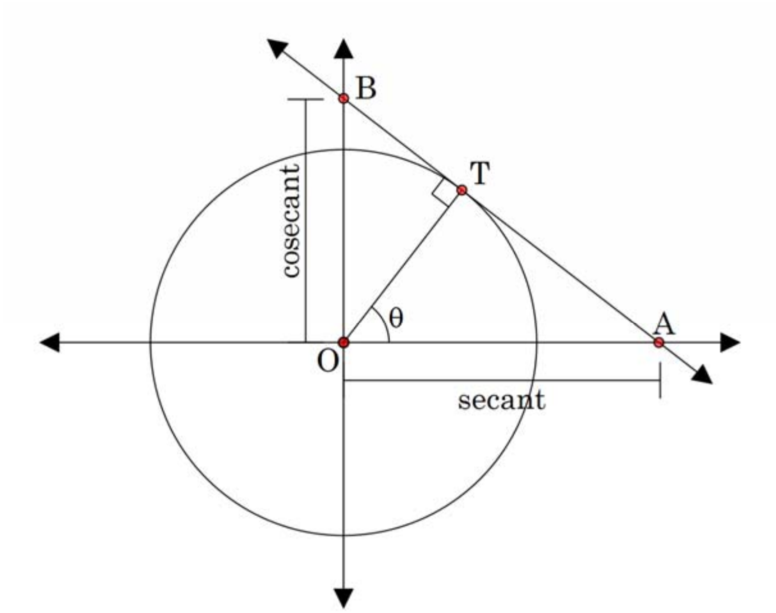
Activity 25 Secant

Aim: Investigate the secant ratio and its associated graph.

Definitions

Consider a unit circle with radial line segment drawn to a point of tangency. As angle θ changes, the intersection points, A and B, of the tangent line and the horizontal and vertical axes move accordingly.

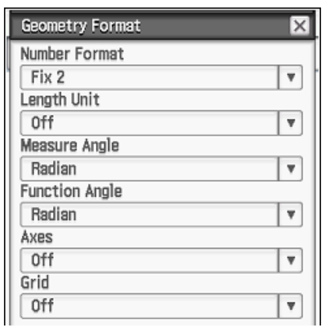


The secant and cosecant of the angle θ are defined by the positions of points A and B as shown above. See Learning Notes for alternative definitions.

We will construct a dynamic unit circle to investigate the secant function.

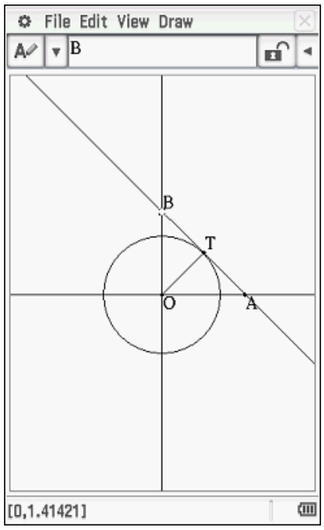
Setup

- Open Main
- Select [⚙ | Geometry Format]
- Modify the settings as shown
- Tap Set



Construct the diagram

- Open Geometry
- Draw a circle, centre (0,0) with radius 1 unit
- Draw a vertical line and change its equation to $x = 0$
- Draw a horizontal line and change its equation to $y = 0$
- Construct a tangent to the circle in the first quadrant
- Draw a radial line segment
- Construct the intersection ⊠ of the tangent line with the horizontal and vertical lines
- Change the labels to match the screenshot



The secant of angle θ, abbreviated to $\sec(\theta)$, is given by the horizontal position of point A.

1. As angle θ changes from 0 to $\dfrac{\pi}{2}$ predict the change in values of $\sec(\theta)$.

2. For what angles will $\sec(\theta)$ be negative?

Add Animation

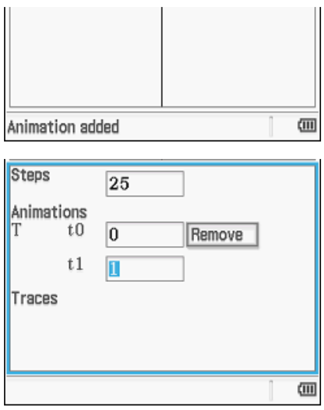
- Select point T and the circle
- [Edit | Animate | Add Animation]

Edit Animation

- [Edit | Animate | Edit Animations]
- Modify the settings as shown

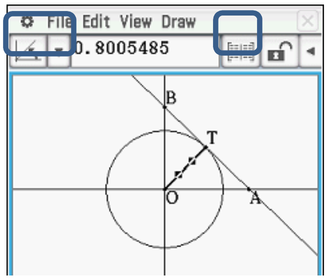
Run the Animation

- [Edit | Animate | Go (once)]



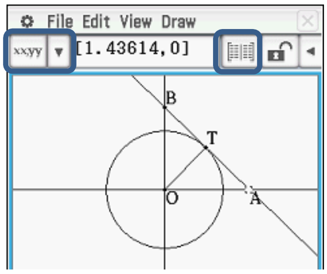
Measure the angle

- Select segment OT
- Select the angle measure from the Measure pull-down menu
- Tap the table button



Measure the secant

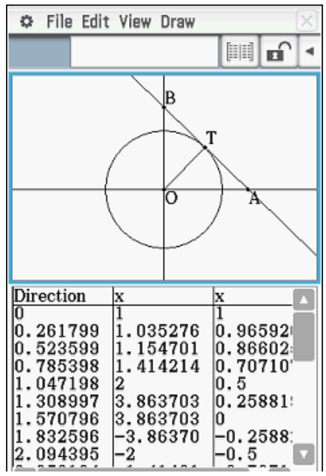
- Select point A
- In the Measure pull-down menu ensure co-ordinates xxyy are selected
- Tap the table button
- Select column y then [Edit | Delete]



Measure the cosine

- Select point T
- Tap the table button
- Select column y then [Edit | Delete]

You should have three columns of data: the angle θ, the secant values $\sec(\theta)$ and the cosine values $\cos(\theta)$ respectively from left to right.



3. Consider the table of values.

   a) What is the incremental change in angle?

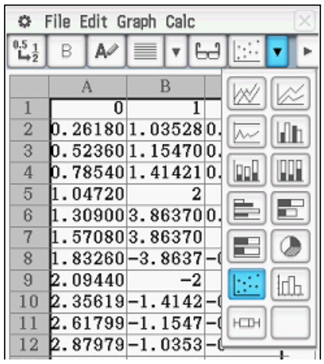
   b) Look at the secant and cosine values for $\frac{\pi}{3}$. What is the relationship between these values?

   c) Look at the secant and cosine values for $\frac{2\pi}{3}$. What is the relationship between these values?

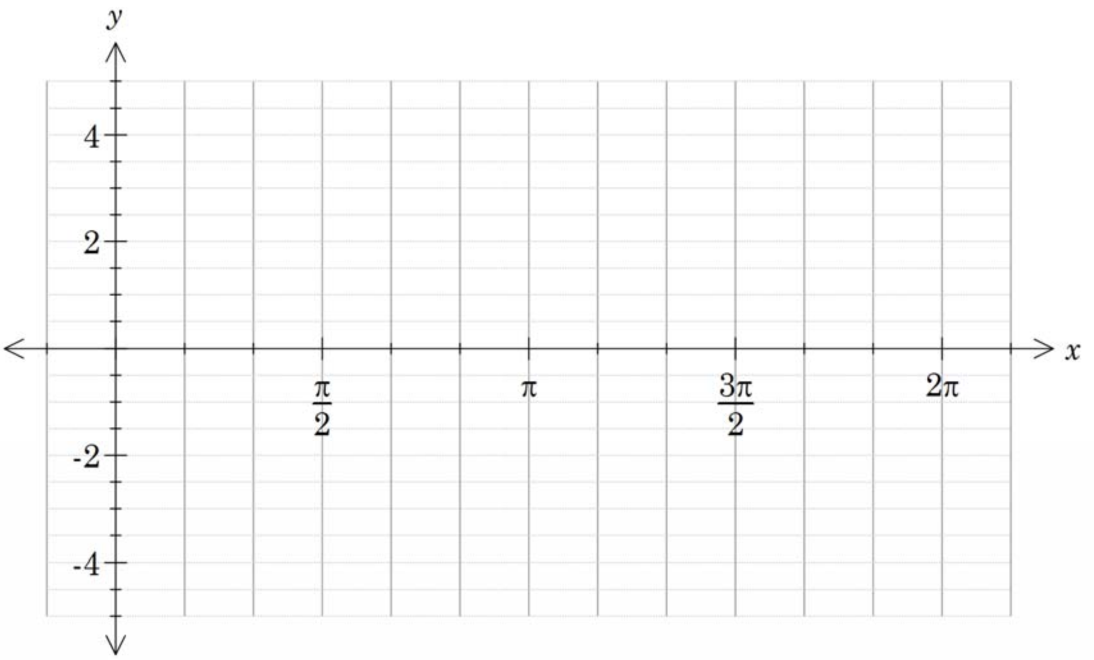
   d) Suggest the relationship between $\sec(\theta)$ and $\cos(\theta)$.

Graph the data

- Select the three columns of data
- [Edit | Copy]
- Open the Spreadsheet application
- [File | New] if necessary
- [Edit | Paste] the values
- Draw a scattergraph of the data

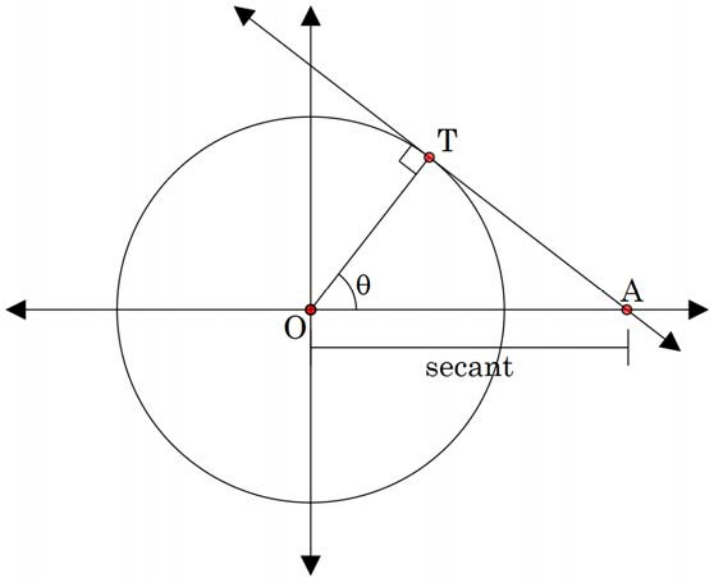


4. Draw a neat sketch of the cosine and secant graphs on the axes below, noting key features of roots and asymptotes. Beware: there are erroneous values for $\sec(\theta)$ at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.



5. Given your answer to Q3 d) explain why $\sec(\theta)$ is undefined for $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$.

6. Consider again the unit circle definition of secant. Note that OT = 1 unit.



a) Explain how the diagram implies the relationship $\sec(\theta) = \dfrac{1}{\cos(\theta)}$.

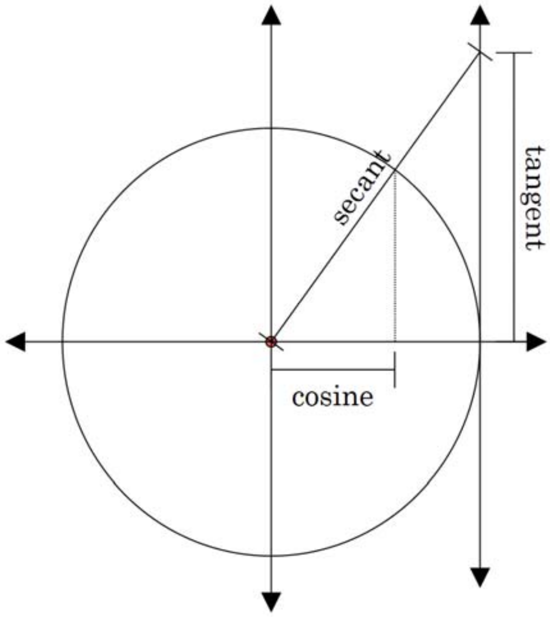
b) Explain why AT = $\tan(\theta)$

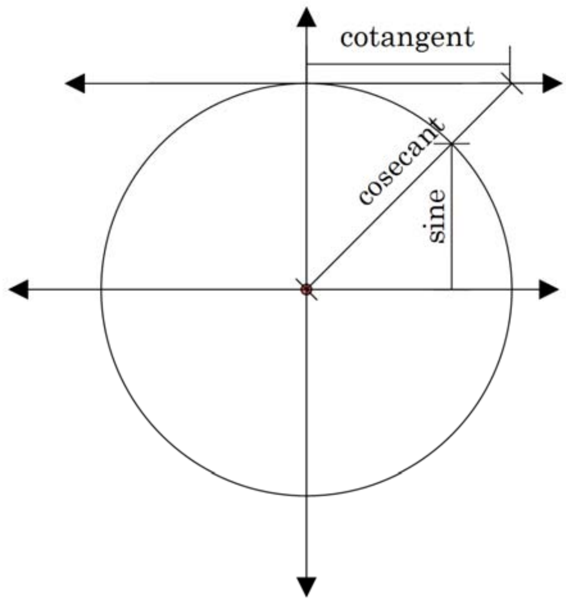
c) Use Pythagoras to write an identity linking $\sec(\theta)$ and $\tan(\theta)$.

d) Prove the identity using the reciprocal relationship in a).

Learning notes

The unit circle definitions of secant and cosecant given at the beginning of this activity are useful for the dynamic unit circle construction. However, more common definitions are shown in the diagrams below, along with the cotangent ratio.





The Geometry construction from this activity will be useful in the next activity to investigate the cosecant ratio. [File | Save] and choose an appropriate name.